The equations and relationships obtained above result in known dependences in a number of particular cases [2—5, 9, 10].

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ON THE ASYMPTOTICS OF UNSTEADY MOTION OF GAS SUBJECTED TO A MOMENTUM

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The motion of gas which initially fills the whole space and is subjected to an instantaneous liberation in a thin layer of initial internal energy E_0 and momentum I_0 is considered. The asymptotic behavior of solution for various relations between E_0 and I_0 is investigated numerically.

When solving unsteady problems of gasdynamics it is often interesting to investigate the asymptotic properties of motion, which are determined for a fairly long time t and are independent of initial data details. In the majority of cases these properties are defined by self-similar solutions. The transition of the flow to the self-similar mode can be traced by solving the exact problem with initial and boundary conditions for the input Euler equations.

Let us consider the plane motion of a perfect inviscid gas free of thermal conductivity for the following initial conditions. Let constant pressure $p = p_0$ and velocity $v = v_0$ be specified for $t = 0$ in a gas layer $0 \leqslant x \leqslant x_0$ (x_0 is the Euler's space coordinate), while in the remaining part of space $p = 0$ and $v = 0$. At the initial instant the density is constant throughout the space and is equal ρ_0. Expressions for the kinetic energy K_0 generated by momentum I_0 and internal energy E_0 specified at the initial instant per unit area for a perfect gas can be written as

$$K_0 = \frac{\rho_0 v_0^2 x_0}{2}, \quad E_0 = \frac{p_0 x_0}{\varkappa - 1}$$

where $\varkappa$ is the ratio of specific heats. We shall consider the asymptotic behavior of solution of the indicated problem for considerable t, when the size of the perturbation motion region $X \gg x_0$. The emergence of an asymptotic solution is evidently determined to a considerable extent by the relation between E_0 and K_0.

In the limit case $K_0 = 0$, $E_0 = \text{const}$ and $x_0 \to 0$ the considered problem reduces to the singular Cauchy problem that defines the explosion of a plane charge in gas [1]. For $E_0 \gg K_0$ the considered motion can be expected to become directly asymptotic that corresponds to the self-similar solution of the problem of a strong explosion, with the time of passing to the limit mode decreasing with increasing E_0 and decreasing x_0 When $K_0 \gg E_0$ and $t \to \infty$ the asymptotics of motion are again defined by the solution of the problem of strong explosion. However, on the other hand, a rarefaction flow will exist for some time to the left of the explosion layer, since at the initial instant gas particles in the layer have an initial velocity v_0 in the direction of the x-axis. A similar situation is a feature of the problem of energy release at the interface of two media of different densities [2]. A lack of symmetry in energy distribution between gas particles moving to the left and right exists at the initial instant. It can be, consequently, expected that during a certain interval of time the shock wave moving to the right will conform to the law of the self-similar solution of the problem of the short shock [3, 4]. A redistribution of energy between the left- and right-hand half-planes (relative to the point of zero velocity) takes place in the course of time; it depends on the flow mode that is established by the strong explosion.

In order to confirm the above reasoning of a qualitative kind, and for a more detailed investigation of the effect of parameters K_0 and E_0 on the asymptotics of the plane motion of gas, computations were carried out for $t = 0$ and $0 \leqslant x \leqslant 0.5$ and the following discontinuous initial data:

$$1^\circ\ p = 2,\ v = 0,\ \rho = 1; \qquad 3^\circ\ p = 1,\ v = \sqrt{5},\ \rho = 1$$
$$2^\circ\ p = 0,\ v = 3.16,\ \rho = 1; \qquad 4^\circ\ p = 1.5,\ v = \sqrt{2.5},\ \rho = 1$$

At all remaining points of space $p = v = 0$ and $\rho = 1$ were assumed at the initial instant, and $\varkappa = 7/5$ was used in computations. The initial data were chosen so that in all cases the total energy $\varepsilon = E_0 + K_0 = \text{const}$. All dependent and independent variables appearing in equations as well as the initial data are taken as dimensionless quantities, with the x-coordinate related to x_0, velocity to v_0, density to ρ_0, pressure to $\rho_0 v_0^2$, and time to x_0 / v_0. These dimensionless quantities leave Euler's equations invariant.

The solution of Euler's equation which defines the considered motion with discontinuous initial data was obtained by the ripple-through method proposed in [5] which makes computation possible throughout the region of motion, including that of shock waves, by the introduction of artificial viscosity.

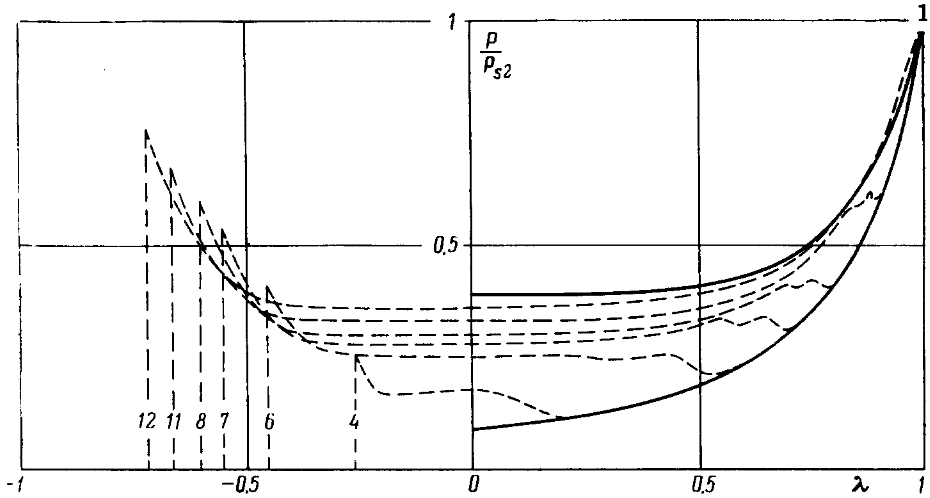

Fig. 1

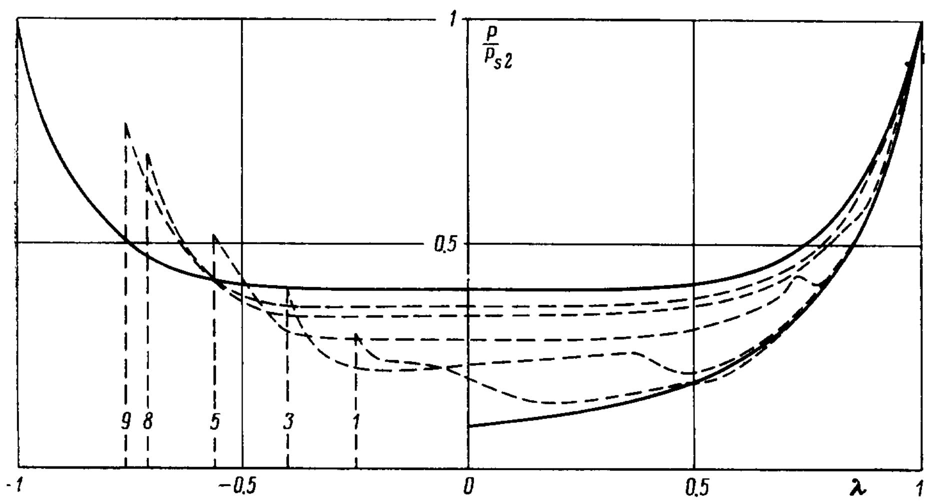

Fig. 2

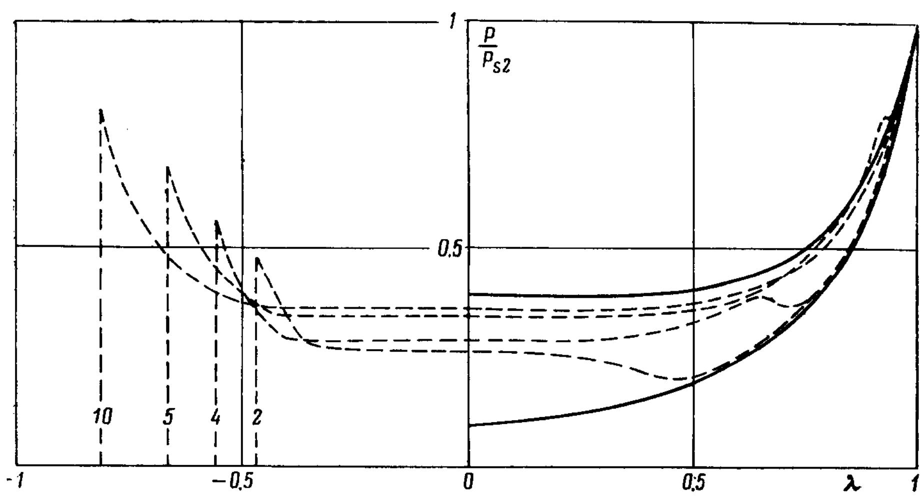

Fig. 3

Computation shows that in Case 1° the solution begins to show fairly quickly (for $t \sim 40$) the asymptotic properties associated with the problem of strong explosion.

The cases 2° — 4°, for which the results of calculations are shown, respectively, in Figs. 1— 3, are more interesting. Curves of pressure distribution p / p_{s2} along the coordinate $\lambda = x / x_{s2}$ are shown in these diagrams. The pressure p_{s2} immediately behind the shock wave moving in the direction of the x-axis and its coordinate x_{s2} were determined in the course of computation. The dash-line curves denoted by numerals 1— 12 relate to the following instants of time t :

N	1	2	3	4	5	6	7	8	9	10	11	12
t	3	5	7	10	28	41	95	150	203	210	263	456

The lower and upper solid curves relate to the self-similar solutions of problems of short shock and strong explosion, respectively.

It will be seen that in the case 2° ($E_0 = 0$ and $K_0 = \varepsilon$) the solution has two asymptotic modes (Fig. 1). The flow that corresponds to the self-similar solution of the problem of short shock obtains in the region of the variable λ which adjoins the right-hand shock wave during some finite but fairly long interval of time ($10 < t < 260$). Region λ in which the flow with short shock obtains changes with time, decreasing with increasing t. A further increase of time ($t > 260$) the indicated intermediate mode begins to change, and the solution for $t \to \infty$ has the asymptotic properties associated with the problem of strong explosion.

The self-similar solution of the problem of short shock is accurate within the constant A which determines the shock wave law of motion $x_{s2} = (At)^{2/5}$. That constant contains some information about initial data and can be determined only by the solution of the non-self-similar problem. The related constant appearing in the law of shock wave motion with strong explosion depends only on the explosion energy ε and the initial density ρ_0 and can be computed in the course of solving the self-similar problem.

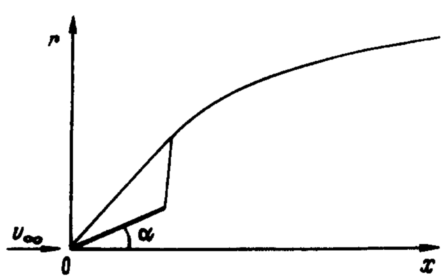

Fig. 4

The pressure distribution shown in Fig. 2 relates to Case 3° ($K_0 = E_0$). It will be seen that here the asymptotics associated with a strong explosion is reached earlier than in the preceding case, nevertheless the asymptotics associated with the short shock appears in the neighborhood of the right-hand shock wave for a time interval that is still fairly long.

The behavior of solution in Case 4° in which $K_0/E_0 = 1/3$ (Fig. 3) is somewhat unexpected. The mode of flow for which the right-hand shock wave moves in conformity with the law of the self-similar solution of the problem of short shock is present here, although the time interval for which that mode occurs is comparatively short ($5 < t < 20$).

It is shown in [6— 9] that one-dimensional unsteady flows may be used for solving the problem of hypersonic flow past a wing of infinite span. As an example, let us consider the flow around a finite plate set at some angle of attack α (Fig. 4). Let us select a not too small α. Strictly speaking, the motion of gas displaced by a piston that is removed after having moved for some time at constant velocity corresponds to such flow in the framework of unsteady analogy. However the solution of the Cauchy problem derived above can be used in qualitative investigations, for which the ratio K_0 / E_0 must evidently be of the order of unity.

As was shown above, a finite time interval exists within which the law of shock wave propagation is determined by the solution of the problem of short shock, even for $K_0 / E_0 = 1/3$. Hence it can be expected that at some fairly considerable distance x from the plate leading edge the shape of the shock wave in a steady hypersonic stream is also determined by the solution of the indicated problem, and that the pressure distribution over the surface of the body rapidly reaches the value obtaining in the case of a strong explosion. It is clear that at infinity downstream the shock wave shape and the velocity field in its neighborhood will vary only little from that predicted in the theory of explosion. The effect of the plane lift can be easily taken asymptotically into account for $x \to \infty$ by the introduction of the "directed" explosion concept according to which not only energy, but also momentum are imparted to the gas [10].

It would be interesting to obtain an experimental confirmation of the proposed here hypothesis on the shock wave shape in the case of hypersonic flow past a body at some angle of attack.

In concluding the author thanks O. S. Ryzhov for his constant interest in this work.

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